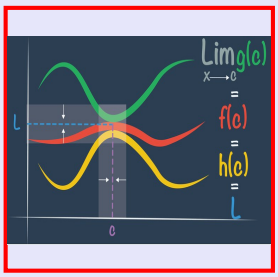
Math 261

Spring 2023

Lecture 55



Feb 19-8:47 AM

Class QZ 17

$$\boxed{f(x) = \int_2^{\sqrt{x}} \frac{1}{t^4+1}\,dt}$$

1) find $f(4) = \int_2^{\sqrt{4}} \frac{1}{t^4+1}\,dt = \int_2^{2} \frac{1}{t^4+1}\,dt = \boxed{0}$ ✓

2) find $f'(x) = \frac{1}{(\sqrt{x})^4+1} \cdot \frac{1}{2\sqrt{x}} - \cancel{\frac{1}{2^4+1} \cdot 0} \to 0$

$= \frac{1}{x^2+1} \cdot \frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}(x^2+1)}}$ ✓

3) find $f'(4) = \frac{1}{2\sqrt{4}(4^2+1)} = \frac{1}{2 \cdot 2(17)} = \boxed{\frac{1}{68}}$ ✓

Eqn. of Tan. line at $x=4$

$y - f(4) = f'(4)(x-4)$

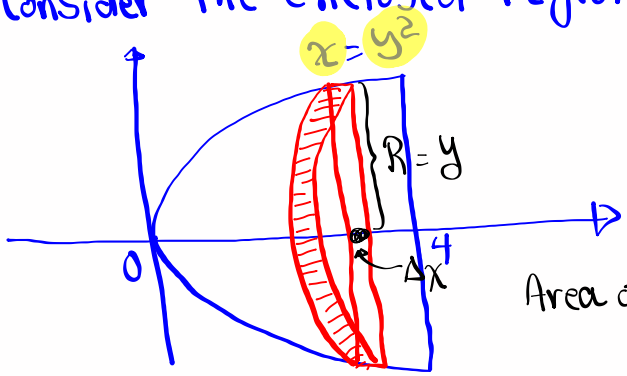
$y - 0 = \frac{1}{68}(x-4)$

$\boxed{y = \frac{1}{68}x - \frac{1}{17}}$

May 23-9:27 AM

Volume of a Solid (No rotation)

Consider the enclosed region below



Cross-Ref. is Semi-Circle $\perp$ x-axis.

Area of Cross-Ref.

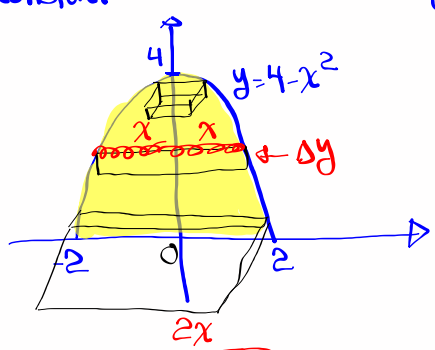
$$\frac{\pi R^2}{2} = \frac{\pi y^2}{2} = \frac{\pi x}{2}$$

$$A(x) = \frac{\pi x}{2}$$

$$V = \int_0^4 A(x)\,dx = \int_0^4 \frac{\pi x}{2}\,dx = \frac{\pi}{2}\cdot\frac{x^2}{2}\Big]_0^4 = \frac{\pi}{4}(4^2 - 0^2) = \boxed{4\pi}$$

May 24-8:51 AM

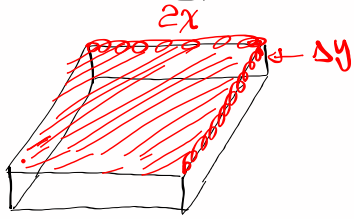
Consider the enclosed region below



Cross-Section is a square and $\perp$ Y-axis.

$$A(x) = 2x \cdot 2x = 4x^2$$

$$V = \int_0^4 4x^2\,dy$$



$$y = 4 - x^2 \rightarrow x^2 = 4 - y$$

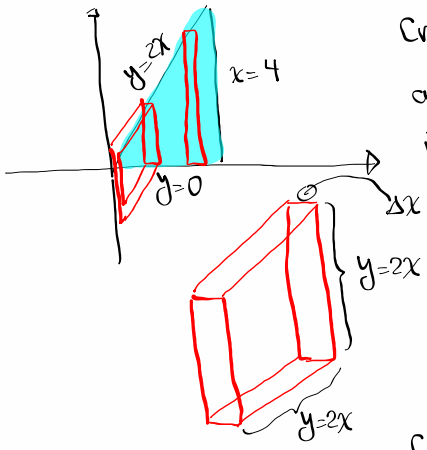
$$V = \int_0^4 4(4-y)\,dy = 4\left[4y - \frac{y^2}{2}\right]_0^4$$

$$= 4\left[\left(4(4) - \frac{4^2}{2}\right) - (0)\right]$$

$$= 4(16 - 8) = 32 \text{ units}^3$$

May 24-8:57 AM

Consider the enclosed region below



Cross-Section ⊥ x-axis.

and each cross-Section is a Square.

$$V=\int_0^4 A(x)\cdot dx$$

$$V=\int_0^4 (2x)^2 dx$$

$$=\int_0^4 4x^2 dx = 4\cdot\frac{x^3}{3}\Big]_0^4$$

$$=\frac{4\cdot 4^3}{3}=\boxed{\frac{256}{3}}$$

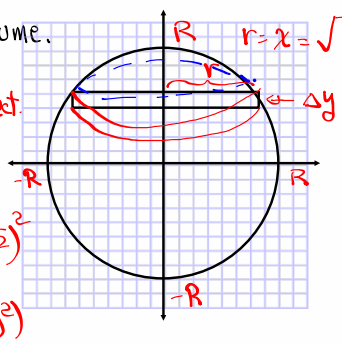
units^3

May 24-9:05 AM

Consider the enclosed region by $\boxed{x^2+y^2=R^2}$. Circle $(0,0)$ Radius R

Cross-Sections are Semi-Circle and ⊥ Y-axis.

Find the Volume.



$r = x = \sqrt{R^2-y^2}$

Area of Cross-Sect.

$$\frac{\pi r^2}{2}$$

$$=\frac{\pi}{2}\left(\sqrt{R^2-y^2}\right)^2$$

$$=\frac{\pi}{2}(R^2-y^2)$$

$$V=\int_{-R}^{R}\frac{\pi}{2}(R^2-y^2)\,dy=\frac{\pi}{\cancel{2}}\cdot\cancel{2}\int_0^R (R^2-y^2)\,dy$$

$$=\pi\left(R^2y-\frac{y^3}{3}\right)\Big]_0^R=\pi\left[\left(R^2\cdot R-\frac{R^3}{3}\right)-(0)\right]$$

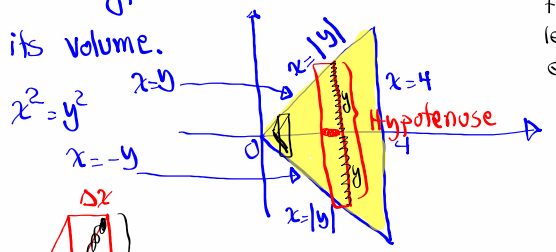
$$=\pi\left(R^3-\frac{R^3}{3}\right)=\boxed{\frac{2\pi R^3}{3}}$$

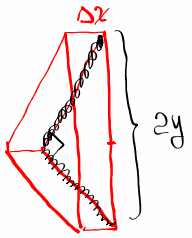
May 24-9:11 AM

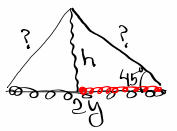
Consider the enclosed region by $x=|y|$ and $x=4$. Cross-Section is a right triangle with hypotenuse $\perp$ to x-axis. Find its volume.

two-legs equal

$x^2=y^2$





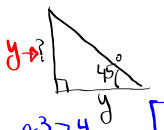


$A=\frac{bh}{2}$

$=\frac{2y\cdot y}{2}$

$=y^2$

$V=\int_0^4 A(x)\,dx$



$=\int_0^4 y^2\,dx=\int_0^4 x^2\,dx=\frac{x^3}{3}\Big]_0^4=$ $\boxed{}$

May 24-9:21 AM

Class QZ 18

Evaluate $\displaystyle\int_0^R x\sqrt{R^2-x^2}\,dx$, $R\geq 0$

$u=R^2-x^2$

$du=-2x\,dx$

$\frac{du}{-2}=x\,dx$

Exact answer only.

All work must be clearly displayed.

$x=0\rightarrow u=R^2$

$x=R\rightarrow u=0$

$$\int_0^{R^2} x\sqrt{R^2-x^2}\,dx=\int_{R^2}^{0}\sqrt{u}\cdot\frac{du}{-2}=-\frac{1}{2}\int_{R^2}^{0}u^{1/2}\,du$$

$$=-\frac{1}{2}\cdot-\int_0^{R^2}u^{1/2}\,du=\frac{1}{2}\cdot\frac{u^{3/2}}{3/2}\Big]_0^{R^2}=\frac{1}{3}u\sqrt{u}\Big]_0^{R^2}$$

$$=\frac{1}{3}\left[R^2\sqrt{R^2}-0\sqrt{0}\right]=\boxed{\frac{R^3}{3}}$$

May 24-9:32 AM